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A REGIONAL AND NON-STATIONARY MODEL FOR PARTIAL DURATION SERIES OF EXTREME RAINFALL

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25 ABSTRACT

26 Regional extreme value models for estimation of extreme rainfall intensities are widely applied, but
27 their underlying assumption of stationarity is challenged. Many recent studies show that the rainfall
28 extremes worldwide exhibit a non-stationary behavior. This paper presents a spatio-temporal model
29 of extreme rainfall. The framework is built on a Partial Duration Series approach with a non-
30 stationary, regional threshold value. The model is based on Generalized Linear Regression solved

31 by Generalized Estimation Equations. It allows a spatial correlation between the stations in the
32 network and accounts furthermore for variable observation periods at each station and in each year.
33 Marginal regional and temporal regression models solved by Generalized Least Squares are used to
34 validate and discuss the results of the full spatio-temporal model.

35 The model is applied on data from a large Danish rain gauge network for four durations ranging
36 from 10 minutes to 24 hours. The observation period differs between stations, and the number of
37 stations with more than 10 years of observations has increased over the years. A spatio-temporal
38 model for the threshold is suggested, applying the Mean Annual Precipitation and time as the
39 explanatory variables in the regional and temporal domain, respectively. Further analysis of Partial
40 Duration Series with non-stationary and regional thresholds shows that the mean exceedances also
41 exhibit a significant variation in space and time for some rainfall durations, while the shape
42 parameter is found to be constant.

43 1 INTRODUCTION

44 Non-stationary extreme value frameworks for estimation of design rainfall or design floods have
45 received increased attention along with the numerous reports of observed trends in extreme rainfall
46 and flood statistics [e.g. *Min et al.*, 2011; *Westra et al.*, 2013; *Sun et al.*, 2015; *Madsen et al.*, In
47 Review]. Regional extreme value models for extreme rainfall have been applied in many countries,
48 e.g. Denmark, Norway, UK, Canada and Australia [*Madsen et al.*, 2002; *Dyrddal et al.*, 2014;
49 *Fowler and Kilsby*, 2003; *Wallis et al.*, 2007; *Haddad et al.*, 2011]. Several frameworks for regional
50 modelling exist, built on different principles. The most traditional is the index-flood model, which
51 became widely applied after the advancement of the regional L-moment approach by *Hosking and*
52 *Wallis* [1993]. In this framework the uncertainty of the estimated at-site design events is reduced by
53 the estimation of regional extreme value distribution (EVD) parameters from a homogenous region.

54 The index-flood model was originally applied for annual maxima extremes in a stationary setting.
55 Later advancements include an extension to Peak over Threshold (POT) data [*Haddad et al.*, 2011;
56 *Madsen and Rosbjerg*, 1997] based on Generalized Least Squares (GLS) estimation procedures
57 [*Stedinger and Tasker* 1985] where the variability of the EVD parameters is modelled by regional
58 covariates. This framework accounts for both at-site sampling uncertainty and spatial dependencies
59 between the observations at different sites. Regarding non-stationarity of the parameters of the
60 EVD, *Gregersen et al.* [2013a] presented a model for the spatio-temporal variation of the number of
61 extreme events building on the framework of Generalized Linear Models (GLM), while *Roth et al.*
62 [2012] used the POT-threshold as the index variable and allowed it to vary in both time and space.
63 As discussed by *Kysely et al.* [2010], a non-stationary threshold is crucial when having a non-
64 uniform distribution of extreme events over the observation period, because the otherwise variable
65 intensity of the Poisson process will violate the asymptotic properties of the POT model. Suitable
66 non-stationary threshold models are often found by quantile regression [*Kysely et al.*, 2010; *Roth et*
67 *al.*, 2012].

68

69 A drawback of the regression based index flood method is that inferences are made on statistics
70 derived from the observations and not from the observations directly [*Renard*, 2011]. Some authors
71 prefer another type of regional model based on hierarchical principles, which overcomes this
72 problem. In the hierarchical model prior distributions are defined for each of the parameters in the
73 EVD [e.g. *Aryal et al.*, 2009; *Dyrddal et al.*, 2014; *Ghosh and Mallick*, 2011; *Heaton et al.*, 2011;
74 *Renard*, 2011; *Sun et al.*, 2015]. From here there are numerous possibilities with regard to the
75 specific model formulation and the parameter estimation algorithms. Hierarchical extreme value
76 models are often solved by sampling algorithms like the Markov Chain Monte Carlo method
77 [*Cooley*, 2007; *Dyrddal et al.*, 2014; *Ghosh and Mallick*, 2011; *Renard*, 2011]. Recently, an

78 alternative approach based on the max-stable framework solved by composite likelihood has gained
79 attention [*Thibaud et al.*, 2013; *Westra and Sisson*, 2011]. *Roth et al.* [2012] also applied composite
80 likelihood but on an index-flood model derived from POT data. Like the hierarchical model, both
81 models can include regional and temporal variability in a dataset with a spatial correlation structure.

82 Regarding regional variations in the parameters some authors use a Kriging technique [*Aryal et al.*,
83 2009], a trivariate Gaussian process with an exponential, distance depending covariance function
84 [e.g. *Cooley*, 2007; *Heaton et al.*, 2011] or a combination of the latter and regression to regional
85 covariates [e.g. *Dyrddal et al.*, 2014; *Ghosh and Mallick*, 2011; *Renard*, 2011). Temporal variations
86 are also assessed together with their influence on the regionalization [e.g. *Aryal et al.*, 2009; *Ghosh*
87 *and Mallick*, 2011; *Heaton et al.*, 2011; *Renard*, 2006; *Roth et al.*, 2012; *Sun et al.*, 2015; *Westra*
88 *and Sisson*, 2011]. Some of these studies include spatial dependencies between the observations at
89 different locations. This can be numerically and theoretically difficult, but with a dense network the
90 uncertainty of the model parameters is greatly underestimated, if the joint probability of observing a
91 given extreme at several sites is disregarded [*Roth et al.*, 2012; *Westra and Sisson*, 2011]. Joint
92 probabilities can be described by copula functions, which therefore can be included in the
93 hierarchical models to account for the spatial dependencies [*Ghosh and Mallick*, 2011; *Renard*,
94 2011]. *Sun et al.* [2015] developed a description applicable for a network of flow measurement
95 stations along a branched river. In a max-stable process for extreme rainfall, spatial dependence on
96 the data level is accounted for by a storm profile model, which is naturally incorporated in the
97 framework [*Thibaud et al.*, 2013; *Westra and Sisson*, 2011].

98

99 The salient point in all above-mentioned non-stationary approaches is the description of the spatial
100 dependencies and, most importantly, the assumptions enforced to solve the complicated equation

101 structure it entails. This becomes particularly difficult for a model based on POT data from a rain
102 gauge network with a varying number of annually active stations, though *Ghosh and Mallick*
103 [2011], *Roth et al.* [2012] and *Thibaud et al.* [2013] show very promising results.

104 The purpose of this paper is to extend the regression based index flood model solved by GLS
105 [*Madsen and Rosbjerg*, 1997] that is currently applied in Denmark for regional estimation of
106 extreme rainfall [*Madsen et al.*, 2002; 2009; In Review) to include a temporal dimension. The
107 procedure developed comprises a GLM solved by the Generalized Estimation Equations (GEE)
108 approach applied by *Gregersen et al.* [2013a] but now extended to more parameters of the EVD.
109 Preliminary analyzes of the temporal and regional variations are performed by marginal models as
110 in *Gregersen et al.* [2013a]. The first analysis aims to formulate a non-stationary threshold model.
111 The quantile regression approach is inapplicable due to the independence criteria traditionally
112 applied for this type of rainfall data (see Section 3 for details). Subsequently, the threshold model is
113 validated and applied. Finally, regional and temporal variations of EVD parameters are assessed.
114 All calculations are performed for four different rainfall durations relevant for urban drainage
115 design.

116 2 METHODOLOGY

117 2.1 PARTIAL DURATION SERIES AND REGIONAL ESTIMATION OF EXTREME RAINFALL

118 The extreme value analysis follows the theory of Partial Duration Series (PDS) where the annual
119 number of extreme events (N) is assumed to follow a Poisson distribution with rate parameter (λ),

$$P\{N = n\} = \frac{\lambda^n}{n!} e^{-\lambda}, \quad N = 0, 1, 2, \dots \quad (1)$$

120 and the magnitude of the extreme events (z) is assumed to follow a Generalized Pareto Distribution
121 (GPD) with a location (β), a scale (α) and a shape (κ) parameter

$$F(z) = P\{Z \leq z\} = 1 - \left(1 + \kappa \frac{z - \beta}{\alpha}\right)^{1/\kappa} \quad (2)$$

122 [Coles, 2001; Rosbjerg *et al.*, 1992]. When sampling the extreme events from a time series, the PDS
 123 approach offers two censoring methods [Mikkelsen *et al.*, 1995]. In type 1 censoring, the threshold
 124 (z_0) over which an event is considered as extreme is pre-fixed. The method is also known as Peak
 125 over Threshold [Coles, 2001]. In this case z_0 replaces β in the GPD in Eq. (2). All other parameters
 126 in Eq. (1) and Eq. (2) are estimated from the dataset. In type 2 censoring, λ , and thereby the total
 127 number of extremes during the observation period, is pre-fixed. In this case all parameters in Eq. (2)
 128 are estimated from the dataset.

129

130 The extreme events in the PDS are required to be independent [Coles, 2001]. In the literature there
 131 are at least two common ways of ensuring this. Many studies perform a declustering procedure
 132 based only on the time span between the extreme events [e.g. Ghosh and Mallick, 2011; Roth *et al.*,
 133 2012], while Madsen *et al.* [2002] performed an event-based separation using information on the
 134 dry-weather period. This method is particularly useful for tipping bucket rainfall measurements,
 135 where the observations are stored as events (see Section 3).

136

137 The procedures from Madsen *et al.*, [2002] are applied here. Therefore, two rainfall events are
 138 independent if the dry weather period between them is longer than or equal to the analyzed rainfall
 139 duration. This event-based separation procedure is performed prior to the PDS approach and is
 140 therefore valid independent of the selected PDS censoring. Available for the extreme value analysis
 141 is therefore a series of independent events (Y) represented by the maximum mean intensity of the
 142 event (y) for a given duration, and the start and termination time (t_{start} , t_{end}) for the event for each

143 site, s (see Madsen et al. [2002] for definition of event-based maximum mean intensity). The dataset
 144 includes in total K stations; $s=1,2,\dots,K$. Furthermore the dataset includes M years of
 145 measurements; $i=1,2,\dots,M$. Not all sites have measurements during the entire period; therefore K
 146 varies from year to year $s_i=1,2,\dots,K_i$ while M varies from site to site $i_s=1,2,\dots,M_s$. In addition, $l_{i,s}$
 147 denotes number of observation days in year i at site s .

148 In the regional model by Madsen et al., [2002, 2009, In Review] type 1 censoring is applied using
 149 the same z_0 at all sites in the region. λ of the Poisson distribution is estimated from

$$\hat{\lambda}_s = \frac{N_s}{l_s} \quad (3)$$

150 where N_s is the total number of exceedances in the full observation period l_s at site s . The GPD
 151 parameters are estimated by the method of L-moments [Hosking, 1990]. From the first L-moment
 152 (b_0), which corresponds to the mean, and the second L-moment (b_1) the following parameters are
 153 defined

$$\hat{\mu} = b_0 \quad \text{and} \quad \hat{L}_{cv} = \frac{b_1}{b_0} \quad (4)$$

154 κ of the GPD is directly related to L_{cv}

$$\hat{\kappa} = \frac{1}{\hat{L}_{cv}} - 2 \quad (5)$$

155 while $\hat{\alpha}$ of the GPD is estimated from $\hat{\mu}$ and \hat{L}_{cv} [Hosking, 1990]. Madsen et al. (2002, 2009,
 156 In Review) showed that λ and μ exhibit significant regional patterns, while L_{cv} and thereby κ is
 157 homogeneous over the region. This gives the following estimate of a regional T-year event

$$\hat{z}_{T,s} = z_0 + \hat{\mu}_s \frac{1 + \hat{\kappa}_R}{\hat{\kappa}_R} \left[1 - \left(\frac{1}{\hat{\lambda}_s T} \right)^{\hat{\kappa}_R} \right] \quad (6)$$

158 where index s refers to a site-specific estimate and R to a regional estimate. As discussed in the
 159 introduction, a clear shortcoming of this model is that temporal variations are not accounted for.

160 The first step pursued in this paper is to develop a spatio-temporal model for β , which can be
 161 applied on the series of independent rainfall events. With a method to describe and evaluate this
 162 variability, it is possible to define a model for z_0 , which will ensure that λ does not exhibit
 163 significant variations over time and space. The second step is to apply the threshold model and
 164 analyze the remaining variability in the GDP parameters.

165 2.1.1 A SPATIO-TEMPORAL MODEL FOR β

166 A PDS is defined from Y using type 2 censoring, PDS_2 . The rate parameter of the Poisson
 167 distribution is prefixed ($\lambda = 4$ events/year). The choice of rate parameter is based on experience
 168 from earlier studies (*Madsen et al.*, [2002, 2009]).]. Since the aim is to capture the regional and
 169 temporal variation, the censoring is performed for each year individually. From PDS_2 β is
 170 estimated for each year, i , and each site, s , as the minimum exceedance in a year. A regression
 171 model is constructed for $\beta_{i,s}$ evaluating both regional and temporal variations, following the
 172 procedures described in Section 2.2 and 2.3.

173 2.1.2 TYPE 1 CENSORING APPLYING A TIME-DEPENDENT AND REGIONAL THRESHOLD

174 The model for β found in the first step is applied as a deterministic model for z_0 . On this basis a
 175 new PDS is defined from Y using type 1 censoring with a pre-fixed z_0 that varies in time and space.
 176 In the following this PDS is denoted PDS_I . The properties of PDS_I is validated by assessing the
 177 variability of λ . λ is a random variable in the model, but λ should not exhibit any significant
 178 variations in time and space, since this variability is explained by the variation in z_0 . Therefore a
 179 regional estimate of λ can be applied. μ and L_{CV} are also estimated from PDS_I and their regional and
 180 temporal variation is assessed following the procedures described in Section 2.2 and 2.3.

181

182 In each of the two steps marginal models are obtained by averaging over either time i or space s .
 183 After this a full spatio-temporal model is developed based on GLM solved by GEE as in *Gregersen*

184 *et al.* [2013a]. In both the full model and the marginal models knowledge on the correlation
 185 between the rain gauge stations is required.

186 2.2 MARGINAL MODELS

187 The general procedures behind the marginal GLS models are similar for all the analyzed GDP
 188 parameters: β from PDS_2 and λ , μ and L_{CV} from PDS_I . The following description of the
 189 methodology applied to assess the variation in space and time, respectively, focuses on β . When
 190 relevant, conditions that make the approach different for one of the other variables are highlighted.

191 2.2.1 MARGINAL MODEL FOR VARIATION IN SPACE

192 Site specific estimates of β are obtained from an average over all years of observation at the site

$$\beta_s = \frac{\sum_{i=1}^{M_s} \beta_{i,s}}{M_s} \quad (7)$$

193 The average of a given quantity asymptotically follows a normal distribution, so analyzes of β_s can
 194 be based on linear regression. GLS is applied to include a possible spatial correlation between the
 195 observations and a weighting that reflects the sampling uncertainty, which for a specific site s is
 196 related to the total period of observation l_s or the total number of observations n_s at that location.

197

198 The following regression model is tested

$$\beta_s = \mathbf{X} \mathbf{B}_{\beta_s} + \boldsymbol{\varepsilon}_{\Sigma} \quad (8)$$

199 where \mathbf{X} is a matrix of regional covariates, \mathbf{B}_{β_s} is the regression parameters, and $\boldsymbol{\Sigma}$ is the variance-
 200 covariance matrix of the model residuals ε . The following structure of $\boldsymbol{\Sigma}$ is assumed in the GLS
 201 model of β_s

$$\Sigma = \begin{bmatrix} \sigma_{\varepsilon 1}^2 + \sigma_{\delta}^2 & \sigma_{\varepsilon 1} \sigma_{\varepsilon 2} \rho_{12} & \cdots & \sigma_{\varepsilon 1} \sigma_{\varepsilon K} \rho_{1K} \\ \sigma_{\varepsilon 2} \sigma_{\varepsilon 1} \rho_{21} & \sigma_{\varepsilon 2}^2 + \sigma_{\delta}^2 & & \sigma_{\varepsilon 2} \sigma_{\varepsilon K} \rho_{2K} \\ \vdots & & \ddots & \vdots \\ & & \sigma_{\varepsilon K} \sigma_{\varepsilon s} \rho_{sK} & \\ & & \sigma_{\varepsilon K} \sigma_{\varepsilon s} \rho_{sK} & \\ \sigma_{\varepsilon K} \sigma_{\varepsilon 1} \rho_{K1} & \sigma_{\varepsilon K} \sigma_{\varepsilon 2} \rho_{K2} & \cdots & \sigma_{\varepsilon K}^2 + \sigma_{\delta}^2 \end{bmatrix} \quad (9)$$

202 In the matrix σ_{ε} represents the sampling uncertainty, σ_{δ} the residual uncertainty in the regression
 203 model, and ρ the spatial correlation. σ_{δ} is assumed to be unknown and is estimated along with the
 204 other variables in the regression model. The estimation of ρ is described in the following section,
 205 while σ_{ε} is a function of the variance of β_s and l_s .

206

207 A description of σ_{ε} for each of the analyzed extreme value statistics is given in Appendix A.1. For
 208 β_s , σ_{ε} is estimated as

$$\hat{\sigma}_{\varepsilon s}^2 = \frac{c}{l_s} \quad \text{where} \quad c = \frac{1}{K} \sum_{s=1}^K \left(\frac{\sum_{i=1}^{M_s} (\beta_{i,s} - \bar{\beta}_s)^2}{M_s - 1} \right) \quad (10)$$

209 where $\bar{\beta}_s$ is the average over all sites.

210

211 A similar procedure is applied for λ_s and μ_s , while for L_{CVs} Monte Carlo simulations are applied (see
 212 Appendix A.1). All estimation procedures of σ_{ε} assume stationarity over time.

213

214 For comparison, ordinary linear regression (OLS) is also applied for the estimation of \mathbf{B}_{β_s} . In OLS
 215 both sampling uncertainty and spatial correlation are disregarded. Furthermore, the statistical
 216 significance of the explanatory variables is evaluated by a t-test on the ratio between the regression
 217 slope and its standard deviation. A similar quantity can be estimated for GLS, but here it is also
 218 important to assess σ_{δ} both in the regression model and in an intercept model, i.e. where only the

219 intercept is included in Eq.(8). If σ_δ is close to zero in the intercept model or negative, all variability
220 is accounted for by the estimated sampling uncertainty, and explanatory variables cannot contribute
221 to a further reduction. Likewise, the value of σ_δ in the regression model can be used to assess the
222 amount of unexplained variability.

223 2.2.2 CORRELATION BETWEEN RAIN GAUGE STATIONS

224 A framework that estimates the spatial dependence between extreme rainfall events at individual
225 rain gauge stations has been developed by *Mikkelsen et al.* [1996] for PDS. It is applied to obtain a
226 robust, regional estimation of the correlation structure to be used in Σ , see Eq. (9). To ensure this,
227 the spatial correlation, ρ , of the GPD parameters is first estimated from the data, and subsequently,
228 described by a parametric correlation model. For further discussion, see *Madsen and Rosbjerg*
229 [1997].

230 For β and λ , ρ is estimated from annual time series of $\beta_{i,s}$ for all pairs of stations. If a rain gauge has
231 long periods of malfunction during a year, it will affect the estimate of $\beta_{i,s}$ and hence ρ . Therefore
232 individual station years are removed from the analysis if the total observation period in a given year
233 is less than 300 days. For μ a method is used, which pairs events that in a physical sense can be
234 regarded concurrent. This is done for each pair of stations, based on t_{start} of the events combined
235 with a lag time, which accounts for the travelling time of the weather systems. For details the reader
236 is referred to *Gregersen et al.* [2013b] and *Mikkelsen et al.* [1996]. Finally, the correlogram is
237 estimated by averaging ρ over distance intervals and subsequently described by an exponential
238 model.

239 2.2.3 MARGINAL MODEL FOR VARIATION IN TIME

240 Annual estimates of β are obtained from an average over all stations. Due to the correlation between
 241 the stations in the network a weighted average is applied so the influence of station clusters with
 242 similar properties is reduced

$$\beta_i = \frac{\sum_{s=1}^{K_i} \beta_{i,s} w_s}{K_i} \quad (11)$$

243 The applied weights (w_s) are estimated for each station as one minus the average correlation
 244 coefficient between the given station and all other stations in the network. High weights are given to
 245 stations with low average correlation, and all weights sum to one.

246
 247 As in Section 2.2.1, analyzes of β_i can be based on linear regression, where GLS is applied to
 248 include a weighting that reflects the sampling uncertainty. Temporal correlation is discarded, as it is
 249 assumed that the estimates of β_i are independent from year to year. The sampling uncertainty of β_i
 250 relates to the total period of observation l_i or the total number of observations at the active stations
 251 K_i in year i .

252

253 The following regression model is tested

$$\beta_i = \mathbf{X} \mathbf{B}_{\beta_i} + \boldsymbol{\varepsilon}_{\Sigma} \quad (12)$$

254 where the following structure of $\boldsymbol{\Sigma}$ is assumed in the GLS model of β_i

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{\varepsilon 1}^2 + \sigma_{\delta}^2 & 0 & \dots & \\ 0 & \sigma_{\varepsilon 2}^2 + \sigma_{\delta}^2 & & \\ \vdots & & \ddots & \\ & & & \sigma_{\varepsilon K}^2 + \sigma_{\delta}^2 \end{bmatrix} \quad (13)$$

255 In the matrix σ_{ε} represents the sampling uncertainty and σ_{δ} the residual uncertainty in the regression
 256 model. σ_{δ} is assumed to be unknown and is estimated along with the other variables in the
 257 regression model. σ_{ε} is estimated as

$$\hat{\sigma}_{\varepsilon i}^2 = \frac{c}{l_i} \quad \text{where} \quad c = \frac{1}{M} \sum_{i=1}^M \left(\frac{\sum_{s=1}^{K_i} (\beta_{i,s} - \bar{\beta}_i)^2}{K_{i,eff} - 1} \right) \quad (14)$$

258 where $\bar{\beta}_i$ is the average over all sites.

259

260 The correlation between the stations, which was accounted for in the estimation of β_i , also affects
 261 the variance of β_i as the amount of independent information in the dataset is reduced. This is
 262 accounted for by estimating the effective number of stations each year using [Madsen *et al.*, 1994]

$$K_{i,eff} = K_i \left(1 + (K_i - 1) \bar{\rho}_i \right)^{-1} \quad (15)$$

263 where $\bar{\rho}_i$ is the average correlation between stations active in the given year.

264

265 A description of σ_ε for λ_i and μ_i is given in Appendix A.2. For comparison, ordinary linear
 266 regression (OLS) is also applied for the estimation of $\mathbf{B}_{\beta i}$, where both sampling uncertainty and
 267 spatial correlation are disregarded.

268 2.3 SPATIO-TEMPORAL MODELS

269 The concept of GLM solved by GEE was applied by *Gregersen et al.* [2013a] for the number of
 270 extreme events, N , which follows a Poisson distribution. GLM applies to all distributions within the
 271 exponential family and also allows for the use of a link function (g) mapping the modelled variable
 272 ($\beta_{i,s}$) and the linear relation to the explanatory variable (\mathbf{X}) given by the regression parameters (\mathbf{B}_η)

$$\boldsymbol{\eta} = \mathbf{X} \mathbf{B}_\eta + \boldsymbol{\varepsilon}_\Sigma \quad \text{where} \quad g(\beta_{i,s}) = \boldsymbol{\eta} \quad (16)$$

273 The link function can ensure that restrictions on the sample space are fulfilled, i.e. for $\beta_{i,s}$, $\lambda_{i,s}$ and
 274 $\mu_{i,s}$ only positive estimates are meaningful. The identity link, a log link and an inverse link are
 275 among the commonly used functions [Faraway, 2006].

Several of the probability distributions from the PDS and GLS frameworks belong to the exponential family, i.e. the Gaussian distribution, the Poisson distribution [Gregersen *et al.*, 2013a] and the Gamma distribution, of which the exponential distribution is a special case. The choice of distribution in the GLM framework can be based on prior knowledge on the process generating the data or from standard evaluations of the data and the model residuals. GLM is solved by maximum likelihood procedures. The log-likelihood is derived from the density function, where the link function is included, before the values of \mathbf{B}_η that optimize the likelihood are estimated, either directly or by an iteration scheme, Gregersen *et al.* [2013a].

With the GEE extension [Halekoh *et al.*, 2006; Hardin and Hilbe, 2003] the GLM framework can account for spatial correlation between stations and a weight ($w_{i,s}$) reflecting how many days during a specific year a given station has been active. Gregersen *et al.* [2013a] showed how the correlation matrix is included in the estimation equations for a Poisson model. The division of the observed variance into sampling uncertainty and model uncertainty done by the GLS model cannot be implemented in the same manner here, as σ_e in the marginal regional model is estimated from the temporal variability and vice versa. GLM with the GEE extension is solved by maximum likelihood procedures.

All calculations are performed using the statistical software R, using the packages ‘geepack’ [Halekoh *et al.*, 2006; Højsgaard *et al.*, 2014], ‘doBy’ [Højsgaard and Halekoh, 2014] and ‘lmom’ [Hosking, 2014].

3 DATA AND CASE STUDY

298 Rainfall series from 83 high-resolution tipping buckets are analyzed. The network was established
299 in 1979 and has expanded since then, while a few stations are closed. All stations included in the
300 present analysis have more than 10 years of measurements. When periods of rain gauge malfunction
301 have been taken into account, the total dataset corresponds to 1881 station-years. The data
302 resolution is one minute and 0.2 mm. Rainfall durations of 10, 60, 180 and 1440 minutes are
303 analyzed.

304

305 The network is operated by the Danish Water Pollution Committee and the Danish Meteorological
306 Institute (DMI), who also conduct the quality-control. Data from this network has since 1998 been
307 applied for estimation of design intensities for urban drainage design [*Madsen et al.*, 2002], and the
308 applied model has been updated twice [*Madsen et al.*, 2009; In Review]. Analyses of temporal
309 variation of extreme rainfall by *Gregersen et al.* [2013a; 2013b] were also based on data from this
310 network.

311

312 The most recent model for regional variation of extreme rainfall over Denmark applied two
313 explanatory variables [*Madsen et al.*, In Review] derived from Climate Grid Denmark (CGD). Here
314 λ varies with the Mean Annual Precipitation (MAP) [mm], while μ varies with the mean value of
315 daily rainfall extremes (μ_{CGD}) [mm]. CGD is a gridded dataset of daily precipitation provided by
316 DMI. It has a spatial resolution of 10x10 km² and covers the period 1989-2010 [*Scharling*, 2012].
317 The grid values are estimated from point measurements obtained from the regional network of daily
318 precipitation stations owned by DMI using an inverse distance weighting method. The documented
319 link between λ and MAP is exploited in the present study; as we aim at having a constant λ over
320 space z_0 must vary with MAP.

321

322 *Gregersen et al.* [2013a] tested several teleconnections as explanatory variables for the temporal
323 variation in N , while *Gregersen et al.* [2014] concluded that the natural variability induced by these
324 teleconnections can appear as short term temporal trends that last a few decades.. As the aim of the
325 present study is to evaluate the influence of **non-stationary** data on a regional extreme value model,
326 *time* is used as the only temporal explanatory variable. In this relation *time* is defined as the number
327 of years since 1978. A year is defined as the calendar year January-December. The majority of
328 extreme rainfall events occur in the period May-October [*Gregersen et al.*, 2013b; *Pedersen et al.*,
329 2012]. Therefore, with the use of the calendar year estimated extreme rainfall statistics are
330 independent from year to year; a necessary assumption as mentioned in Section 2.2.3.

331 4 RESULTS

332 4.1 SPATIO-TEMPORAL MODELS FOR β

333 From type 2 censoring PDS_2 is defined as the fourth largest independent event at each site for each
334 year. Subsequently, β is estimated and the regional and temporal variability are evaluated by the
335 two marginal models and the full spatio-temporal model. All models take into account the spatial
336 dependence between the PDS series, which is estimated first.

337 4.1.1 CORRELATION BETWEEN RAIN GAUGE STATIONS

338 The spatial correlation structure, applied in the three models of β is given in Figure 1. The shape of
339 the correlogram depends on the rainfall duration. In general the correlation decreases with distance,
340 but stations far away remains correlated. The shape of the curves corresponds well with the findings
341 of *Gregersen et al.* [2013a] and *Madsen et al.* [In Review].

342 4.1.2 MARGINAL REGIONAL MODEL FOR β

343 Initial analyzes of the intercept model (not shown) confirm that $\hat{\sigma}_\delta^2$ is positive for all durations.
344 Hence regional variability exists, which potentially can be reduced by the introduction of
345 explanatory variables. The following regression model is analyzed by GLS and for means of
346 comparison by OLS:

$$\beta_s = a + b \cdot MAP + \varepsilon_\Sigma \quad (17)$$

347 The estimated model parameters are given in Table 1, while the observations and the modelled
348 relation are shown in Figure 2 for all four durations. Accounting for spatial correlation and
349 sampling uncertainty change the regression parameters and their uncertainty slightly (see Table 1).
350 An influence of *MAP* is suggested for very long rainfall (1440 min) durations.

351

352 As discussed in *Madsen et al.* [In Review], it is expected that the variability over time, when not
353 accounted for, is introducing an additional uncertainty in the regional model. The length of
354 observation period differs between stations. An effect of this can be seen from Figure 2 where
355 stations, which have been active approximately during the entire period from 1979 to 2012, are
356 marked with black. The variability in this subset of stations is remarkably lower than the variability
357 of the entire sample. In Section 4.1.4, it is shown that the spatio-temporal model accounts for
358 this.

359 4.1.3 MARGINAL TEMPORAL MODEL FOR β

360 For β the spatial correlation is relatively high, also for sites separated by long distances (see Section
361 4.1.1). This has a substantial effect on the sampling uncertainty, because $K_{i,eff}$, despite the large
362 number of stations in the network, is very small, see Eq.(15). Still, initial analyzes of the
363 intercept model (not shown) confirm that $\hat{\sigma}_\delta^2$ is positive for 10 and 60 minutes and is greatly
364 reduced by the introduction of temporal regressors. *Gregersen* [2015] showed that when λ is

365 analyzed in a similar manner $\hat{\sigma}_\delta^2$ is positive for all durations. Therefore the analysis is continued for
 366 all durations and the following regression model is analyzed by GLS:

$$\beta_i = a + c \cdot time + \varepsilon_\Sigma \quad (18)$$

367 Again, parameter estimates from OLS regression are also assessed. Estimated model parameters are
 368 given in Table 2, while the observations and the modelled relations are shown in Figure 3 for all
 369 four durations. For all rainfall durations β_i increases over time. A comparison between OLS and
 370 GLS (not included) shows that the standard errors on the regression slopes for 10 and 60 minutes
 371 increase slightly when the sampling uncertainty is taken into account. Besides that the other
 372 parameter estimates are almost identical for the two methods.

373 4.1.4 THE FULL SPATIAL AND TEMPORAL MODEL FOR β

374 The two marginal models show variability in both domains and thereby a need for the formulation
 375 of a spatio-temporal model. The following regression model is analyzed by GLM-GEE

$$\eta_{i,s} = a + b \cdot MAP + c \cdot time + \varepsilon_\Sigma \quad \text{where} \quad g(\beta_{i,s}) = \eta_{i,s} \quad (19)$$

376 It must be evaluated which underlying probability distribution is best suited to describe $\beta_{i,s}$. A
 377 histogram combined with a kernel-approximation of the density function shows no large deviations
 378 from a Gaussian distribution, see Figure 4a. Furthermore, different distributions and link functions
 379 were tested in the GLM-GEE model. It was found that the parameter estimates varied little and that
 380 the residuals showed approximately the same behavior independent of the choice of model, see
 381 Figure 4b and Figure 4c. For the convenience of direct comparison with the parameter estimates
 382 from the two marginal GLS models, the subsequent analysis is based on a GLM-GEE model with a
 383 Gaussian distribution and an identity link function.

384

Table 3 shows the results for all four rainfall durations using the most complex GLM-GEE model with both weights and correlation. It has been evaluated how the assumption on the structure of the residuals (weights and correlation) affects the parameter estimates and their significance (not shown). The conclusion depends on the rainfall durations, but in general the variation of the parameter estimates and their significance are not substantially different from the variation between the OLS and the GLS estimates in the marginal models.

For all durations the full model with weights and correlation reduces the parameter uncertainty on b compared to the marginal regional GLS model, see Table 3. This leads to a significant correlation to MAP for all durations but 60 minutes. The estimate of b has decreased for 10 min, but increased for 180 and 1440 minutes. This corresponds well to the behaviour reported by *Madsen et al.* [In Review]. All in all it is confirmed that when the assumption of stationarity is not fulfilled the overall model uncertainty increases with the available information. The spatio-temporal model solves this issue. The estimates of c are also slightly different from the estimates of the marginal temporal model as an effect of the variability explained by MAP and the covariance between the parameter estimates.

4.2 TYPE 1 CENSORING APPLYING A SPATIO-TEMPORAL THRESHOLD

The spatio-temporal model of β is applied as a deterministic model for z_0 to define a new PDS from Y using type 1 censoring (PDS_I). When z_0 varies in time and space, λ must not exhibit any significant variations over time and space. For a duration of 60 minutes a regression to MAP is included even though it is non-significant.

From PDS_I the GPD parameters from Eq.(6) are estimated, and their variation over time and space is assessed including the spatial correlation structure.

409 4.2.1 CORRELATION BETWEEN RAIN GAUGE STATIONS

410 The spatial correlation structure for λ and μ is given in Figure 5 and Figure 6, respectively. The
 411 shape of the correlogram depends on the rainfall duration. In general, the correlation decreases with
 412 distance, and this happens quickly for μ , while for λ stations far away remain correlated. A
 413 comparison with the findings by *Madsen et al.* [In Review] (not shown) indicates that when a non-
 414 stationary threshold is applied the correlation decreases slightly.

415 4.2.2 VARIATIONS OF λ OVER TIME AND SPACE

416 The two following marginal regression models are analyzed

$$\lambda_s = a + b \cdot MAP + \varepsilon_\Sigma \quad (20)$$

$$\lambda_t = a + c \cdot time + \varepsilon_\Sigma \quad (21)$$

417 to test that the application of the non-stationary and regional z_θ leads to constant λ over time and
 418 space. This is confirmed. Furthermore, the variability of all data points and a subset of data points
 419 representing the stations with long records (black dots) are comparable, see Figure 7. This is not the
 420 case when the temporal variability is not accounted for, see Figure 2. Station outliers are observed
 421 for all durations. Two stations from suburbs west of Copenhagen recur as outliers. As they both are
 422 approved by the quality control, they are retained in the dataset. One of the stations was closed in
 423 1995.

424 The following regression model is analyzed by GLM-GEE to estimate the constant λ and the
 425 uncertainty on the estimate:

$$\eta_{i,s} = a + \text{offset}(\log(l_{i,s})) + \varepsilon_\Sigma \quad \text{where} \quad \log(N_{i,s}) = \eta_{i,s} \quad (22)$$

426 $N_{i,s}$ is assumed to follow a Poisson distribution, while the variability of $\lambda_{i,s}$ can be modelled by
 427 including the observation period in the model as an offset. A Poisson model with a log-link function
 428 is applied following the approach by *Gregersen et al.* [2013a]. The results are given in Table 4.

429 The scale of the Poisson distribution is reported to evaluate if $N_{i,s}$ shows overdispersion; values
 430 close to one indicate that this is not the case. The results are applied for estimation of design
 431 intensities in Section 4.3.

432 4.2.3 VARIATIONS OF μ OVER TIME AND SPACE

433 Initial analyzes of the marginal intercept models (not shown) confirm that $\hat{\sigma}_\delta^2$ is positive for all
 434 durations in both the temporal and regional domain. The following two marginal regression models
 435 are analyzed

$$\mu_s = a + b \cdot \mu_{CGD} + \varepsilon_\Sigma \quad (23)$$

$$\mu_i = a + c \cdot time + \varepsilon_\Sigma \quad (24)$$

436 The regression between μ and *time* is positive, see Figure 8, and significant for all durations but
 437 1440 minutes, see Table 5. The regression between μ and μ_{CGD} is positive, see Figure 9, and
 438 significant on a 10% confidence level for 10, 180 and 1440 minutes, see Table 5. However, residual
 439 analysis (not shown) indicates that the marginal model for the variation in space is too simple,
 440 because the variance seemingly increases with the mean for both 60 and 180 minutes. This tendency
 441 can also be seen directly from Figure 9. The full spatio-temporal variation is described by the
 442 following model and analyzed by GLM-GEE

$$\eta_{i,s} = a + b \cdot \mu_{CGD} + c \cdot time + \varepsilon_\Sigma \quad \text{where} \quad g(\mu_{i,s}) = \eta_{i,s} \quad (25)$$

443 A histogram of $\mu_{i,s}$ shows a distribution with a positive skew, which is suggesting the use of a
 444 Gamma distribution in the GLM-GEE, see Figure 10. A Gamma model is also suitable if the
 445 variance of the residuals increases with the mean. As potential link functions for the Gamma model,
 446 a log link and an inverse link are considered. Evaluations show that the significance of the
 447 regression parameters are almost identical for the two different link functions (not shown), but
 448 residual analysis suggests that the increase of the variance is captured best by the inverse link, see
 449 Figure 10. Therefore, the inverse link is applied. The estimated model parameters are given in Table

450 5. These show a difference between the significance of the regression coefficients in the marginal
 451 models and the full model. The reason is that the underlying model assumptions differ as the
 452 gamma model can account for an increase of the variance. This probably leads to the found
 453 difference in significance of μ_{CGD} for 10 and 180 minutes.

454 4.2.4 VARIATIONS OF L_{CV} OVER TIME AND SPACE

455 L_{CV} exhibits no apparent variation over space and time (not shown). Estimation of the heterogeneity
 456 measure from *Hosking and Wallis* [1993] confirms that the parameter can be regarded as
 457 homogenous. L_{CV} is estimated following the method from *Madsen et al.* [2002], and results are given
 458 in Table 6.

459 4.3 ESTIMATION OF DESIGN INTENSITIES

460 Based on the results from the preceding sections the following model for the estimation of T-year
 461 events is suggested

$$z_{T,i,s} = z_{0,i,s} + \mu_{i,s} \frac{1+\kappa}{\kappa} \left[1 - \left(\frac{1}{\lambda T} \right)^\kappa \right] \quad (26)$$

462 $z_{0i,s}$ is fixed in the applied PDS procedure and hence by definition not uncertain. All other
 463 parameters estimated from PDS_I are predicted from the regional and temporal covariates, if these
 464 are found significant in the full model. By use of Eq.(26) the 10-year event ($z_{10i,s}$) is predicted for
 465 two locations in Denmark and compared to the results from *Madsen et al.* [In Review], see Table 7.
 466 The uncertainty on $z_{10i,s}$ is approximated by a second order Taylor expansion of Eq.(26) [*Madsen et*
 467 *al.*, 2002].

468

469 The temporal development of the 10-year event is strongest for 10, 60 and 180 minutes, where both
 470 λ and μ has a non-stationary component. For these three durations the estimate from *Madsen et al.*

471 [In Review] lies in between the two years for which predictions are made (see Table 7). This
472 corresponds well with the data that the model by *Madsen et al.* [In Review] is built on. For 1440
473 min the variability is strongest in the regional domain as both λ and μ has a regional component. A
474 small temporal variability is seen due to the temporal development in λ . Comparing the estimated
475 uncertainties on the 10-year events, with the results from *Madsen et al.* [In Review] it is seen, that
476 the proposed regional and non-stationary model for PDS successfully eliminates the uncertainty
477 introduced when the assumption of stationarity is not fulfilled.

478 5 DISCUSSION

479 The identified model is primarily used for explorative purposes. Over the last 15 years the urban
480 drainage engineering community has experienced increases of recommended design intensities of
481 extreme precipitation of the ‘present’ (i.e. ‘stationary’) climate that by far exceeds the anticipated
482 changes due to emissions of greenhouse gasses in any of the future scenarios foreseen by the
483 climate change communities. The purpose of the model has been to validate that this process has
484 taken place and to separate the process into a regional and a temporal component. Future work will
485 focus on separating the temporal component into processes that can be validated and predicted,
486 thereby distinguishing between observed dynamics in the past, present, and future, and anticipated
487 climatic changes due to emissions of greenhouse gasses.

488

489 Using the identified model it is in principle possible to obtain design values for infrastructure by
490 extrapolating into the future and consider e.g. the maximum design values over the expected
491 technical lifetime and using that in the design process. Procedures for making such projections
492 (although for different variables than precipitation) are described in e.g. *Larsen et al.* [2003] and
493 *Arnbjerg-Nielsen* [2011]. However, as pointed out in a commentary by *Montanari and*

494 *Koutsoyiannis* [2014] the practical application of statistical models with a time-dependent
495 component can be questionable, which is why process-based models are preferable. This study has
496 not addressed the physical processes which could lie behind the temporal variation. Several studies
497 indicate that large scale climatic variables may influence extreme rainfall [e.g. *Sun et al.*, 2014;
498 *Willems* 2013], but even when the large scale physical drivers are known, several challenges still
499 remain; first to predict their future states, secondly to select which state to apply when choosing the
500 design intensity. Hence, the present study demonstrates the non-stationary model using time as
501 covariate. Since all model applications are uncertain outside the range of the input variables we do
502 not recommend to use the identified model in its present form for estimating design intensities for
503 the future.

504

505 As pointed out by *Willems* [2013] and *Gregersen et al.* [2014] there may be some cyclical processes
506 occurring with low frequency that, once understood, can serve as a more suitable co-variate in the
507 model. This would enable projections into the future that are more trust-worthy if future states of
508 these co-variates can be obtained. If climate drivers replace the variable *time*, this will have an
509 effect on the model residuals and the estimated uncertainty on the design intensities. This potential
510 reduction in uncertainty is, based on earlier work using teleconnections, assumed to be minor in
511 comparison to the difference between including and not including a temporal variable [*Gregersen et*
512 *al.*, 2013a].

513 6 CONCLUSION

514 In conclusion, the suggested GLM-GEE procedure is shown to be highly qualified for modelling
515 spatio-temporal variability in β , λ and μ and thereby for the construction of a non-stationary regional
516 extreme value model, which can be used for the estimation of urban design rainfall.

517

518 The dataset is first assessed by type 2 PDS censoring to find a suitable model for the threshold. The
519 model is subsequently used in type 1 PDS censoring to ensure that the occurrences of extreme
520 events are identically distributed over the sample period. These PDS are analyzed for a temporal
521 and regional development in the mean exceedances and the shape parameter. The GLM-GEE
522 framework can include a spatial correlation between the measurements and a weight reflecting a
523 possible difference in the observation period. However, it is not possible to include the sampling
524 uncertainty of the observation directly. If the magnitude of the sampling uncertainty is equal to the
525 total variability of the data, there is no reason to search for regional or temporal explanatory
526 variables. It is therefore recommended to assess the marginal development in space and time, and
527 the magnitude of sampling uncertainty in both domains before the full spatio-temporal model is
528 applied. Thereby, model overfitting and underestimation of uncertainty are avoided. A GLS
529 framework is well suited for evaluation of the marginal models.

530 Rainfall duration of 10, 60, 180 and 1440 minutes were analyzed. For β , a spatio-temporal model is
531 suggested, applying the Mean Annual Precipitation and time as the explanatory variables in the
532 regional and temporal domain, respectively. This threshold model ensures a constant λ . A
533 significant temporal increase in the mean of the extreme exceedances was found for 10, 60, 180
534 minutes, while for 1440 minutes a significant regional variability was found. No significant
535 temporal or regional variability of the shape parameter was found, and hence a regionally constant
536 estimate was applied.

537

538 *Westra and Sisson* [2011] and *Roth et al.* [2012] assessed the effect of spatial correlation on the
539 identification of temporal trends by different methods. A comparison of methods is highly relevant.
540 The next step is to evaluate the differences and similarities of the proposed model and the methods

541 of *Ghosh and Mallick* [2011], *Roth et al.* [2012] and *Thibaud et al.* [2013], in terms of spatial
542 correlation, significance of spatial and temporal variables, and prediction of urban design
543 intensities.

544

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551 authors can provide both R-scripts and processed data-series for the reproduction of the published
552 results.

553

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675 APPENDICES

676 A.1 SAMPLING UNCERTAINTY FOR THE MARGINAL REGIONAL MODEL

677 The applied procedure for estimation of the sample uncertainty on β_s , λ_s , μ_s and L_{CVs} are reviewed
678 below, starting with β_s . Unless specifically stated the procedures follow *Madsen and Rosbjerg*
679 [1997].

680

681 For any of the of the four GPD parameters the GLS procedure requires, that the at-site estimate of
 682 the sampling uncertainty ($\hat{\sigma}_{\varepsilon s}$) is nearly independent of the estimated parameter ($\hat{\theta}_s$) [*Madsen and*
 683 *Rosbjerg, 1997*]. *Madsen and Rosbjerg* [1997] and *Madsen et al.* [2002] therefore applied an
 684 estimation procedure based on sample estimates of the population parameters.

685

686 A given site s has M_s observations of β ; one for each year where the station has been active. On this
 687 basis the mean and variance of β are estimated for each site, and subsequently for the entire
 688 population by averaging over all K sites. A site specific estimation of the sampling uncertainty is
 689 obtained by accounting for the actual observation period at the site, l_s :

$$\hat{\sigma}_{\varepsilon s}^2 = \frac{c}{l_s} \quad \text{where} \quad c = \frac{1}{K} \sum_{s=1}^K \left(\frac{\sum_{i=1}^{M_s} (\beta_{i,s} - \overline{\beta}_{0s})^2}{M_s - 1} \right) \quad (1A)$$

690 where $\overline{\beta}_s$ is the average over all sites.

691

692 The sampling uncertainty of λ_s is estimated from a procedure identical to the one described for β_s . In
 693 *Madsen and Rosbjerg* [1997] and *Madsen et al.* [2002] it is utilized that λ_s is the rate parameter of
 694 the Poisson distribution and hence the variance equals the mean. To ensure consistency between the
 695 sampling uncertainty estimates in the marginal regional and temporal model the approach from
 696 equation 1A is preferred. In the marginal temporal model correlation between stations is accounted
 697 for in the estimation of the variance. This procedure cannot be applied if the variance is assumed
 698 equal to the mean.

699

700 The sampling uncertainty of μ_s is estimated from a procedure similar to the one described for β_s . A
 701 given site s has n_s observations of extreme rainfall with intensity y . n_s is indirectly related to how
 702 many years the station has been active. The mean and variance are estimated for each site, and

703 subsequently for the entire population by averaging over all K sites. A site specific estimation of the
 704 sampling uncertainty is obtained by accounting for the number of observations at the site, n_s :

$$\hat{\sigma}_{\varepsilon s}^2 = \frac{c}{n_s} \quad \text{where} \quad c = \frac{1}{K} \sum_{s=1}^K \left(\frac{\sum_{j=1}^{n_s} (y_j - \mu_s)^2}{n_s - 1} \right) \quad (2A)$$

705 where μ_s is the average over all sites.

706

707 For higher moments the estimation of the sampling uncertainty cannot be based on the population
 708 parameters if the independence criteria should be fulfilled. Simulations are therefore required. A
 709 four parameter kappa distribution is suitable for this purpose because of its flexibility [*Madsen et*
 710 *al.*, 2002].

711

712 The sampling uncertainty of L_{CVs} is estimated from Monto Carlo simulation using the kappa
 713 distribution with parameters estimated from regional estimates of the first four L-moments. In total
 714 500 series with \bar{n}_s observations are generated from this distribution, and applied for estimation of
 715 $\text{Var}(L_{cv})_{mc}$. Here \bar{n}_s is the average number of events over all sites. A site specific estimation of the
 716 sample uncertainty is obtained accounting for the ratio between \bar{n}_s and the actual number of
 717 observations at the site, n_s :

$$\hat{\sigma}_{\varepsilon s}^2 = \frac{c}{n_s} \quad \text{where} \quad c = \text{Var}(L_{cv})_{mc} \cdot \bar{n}_s \quad (3A)$$

718 A.2 SAMPLING UNCERTAINTY FOR THE MARGINAL TEMPORAL MODEL

719 The applied procedure for estimation of the sampling uncertainty of β_i , λ_i and μ_i is reviewed below,
 720 starting with β_i .

721

722 The marginal temporal GLS model is similar to the marginal regional GLS model, just averaging
 723 over the regional domain instead. The considerations regarding appropriate estimators of the
 724 sampling uncertainty given for the regional GLS model therefore applies. With the important
 725 difference that the estimate of sampling uncertainty is affected by the correlation between sites,
 726 which reduces the amount of information in the data set and increases the sample estimate of the
 727 population variance.

728

729 For β a given year i has K_i observations of β , one for each station active that year. On this basis the
 730 mean and variance are estimated for each year. The variance is corrected for the correlation
 731 between stations by dividing the sum of squared differences with $K_{i,eff}$, which is estimated from:

$$K_{i,eff} = K_i \left(1 + (K_i - 1) \overline{\rho_i} \right)^{-1} \quad (4A)$$

732 where $\overline{\rho_i}$ is the average correlation between stations active in the given year. The mean and
 733 variance are estimated by averaging over all M years. A site specific estimation of the sampling
 734 uncertainty is obtained by accounting for the actual number of active stations in the given year and
 735 their breakdown periods, i.e. the number of station years l_i :

$$\hat{\sigma}_{\varepsilon i}^2 = \frac{c}{l_i} \quad \text{where} \quad c = \frac{1}{M} \sum_{i=1}^M \left(\frac{\sum_{s=1}^{K_i} (\beta_{i,s} - \overline{\beta_{0i}})^2}{K_{i,eff} - 1} \right) \quad (5A)$$

736 The sampling uncertainty of λ_i is estimated from a procedure identical to the one described for β_i .

737

738 The sampling uncertainty of μ_i is also estimated from a procedure similar to the one described for
 739 β_i . A given year i has n_i observations of extreme rainfall with intensity y . On this basis the mean and
 740 variance are estimated for each year. The variance must again be corrected for the correlation
 741 between stations. However, as one site often has more than one observation and at-site observations
 742 are uncorrelated $n_{i,eff}$ is estimated as:

$$n_{i,eff} = \bar{n}_i K_{eff} \quad (6A)$$

743 where $\bar{\rho}_i$ is the average correlation between stations active in the given year and $K_{i,eff}$ is given by
 744 (4A). The mean and variance are estimated by averaging over all M years. A site specific estimation
 745 of the sampling uncertainty is obtained by accounting for the actual number of active observations
 746 that year:

$$\hat{\sigma}_{\varepsilon i}^2 = \frac{c}{n_i} \quad \text{where} \quad c = \frac{1}{M} \sum_{i=1}^M \left(\frac{\sum_{j=1}^{n_s} (y_j - \mu_i)^2}{n_{i,eff} - 1} \right) \quad (7A)$$

747

All figures with caption. Figure

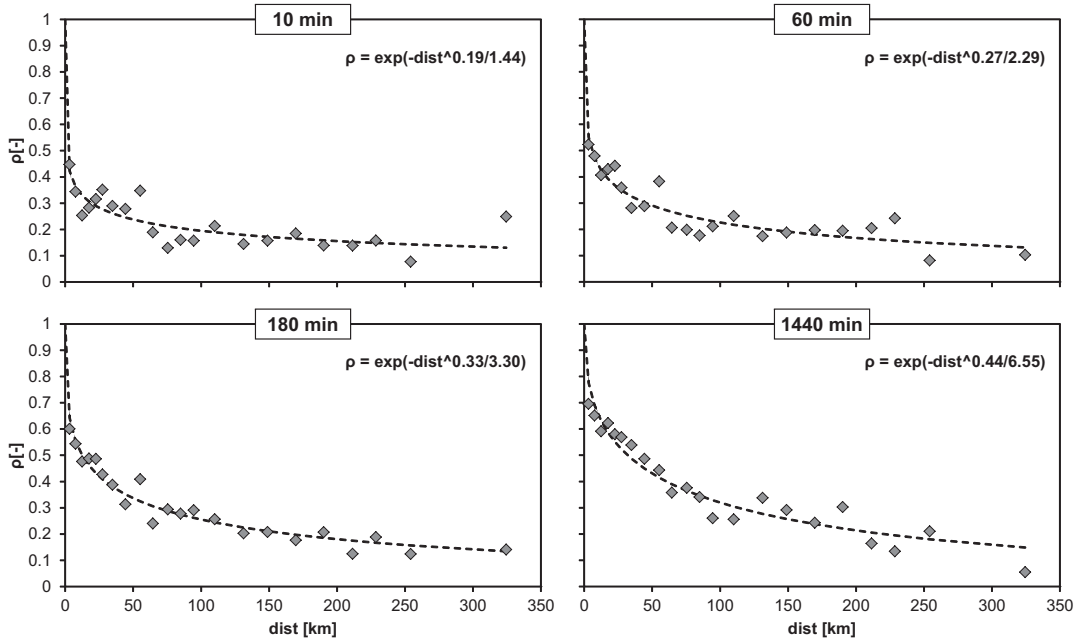


Figure 1. The correlogram for the spatial variation of z_0 for the four durations. Dots represent the observed correlation averaged in selected bins, the black line represents the fitted exponential model given in the top left of each graph.

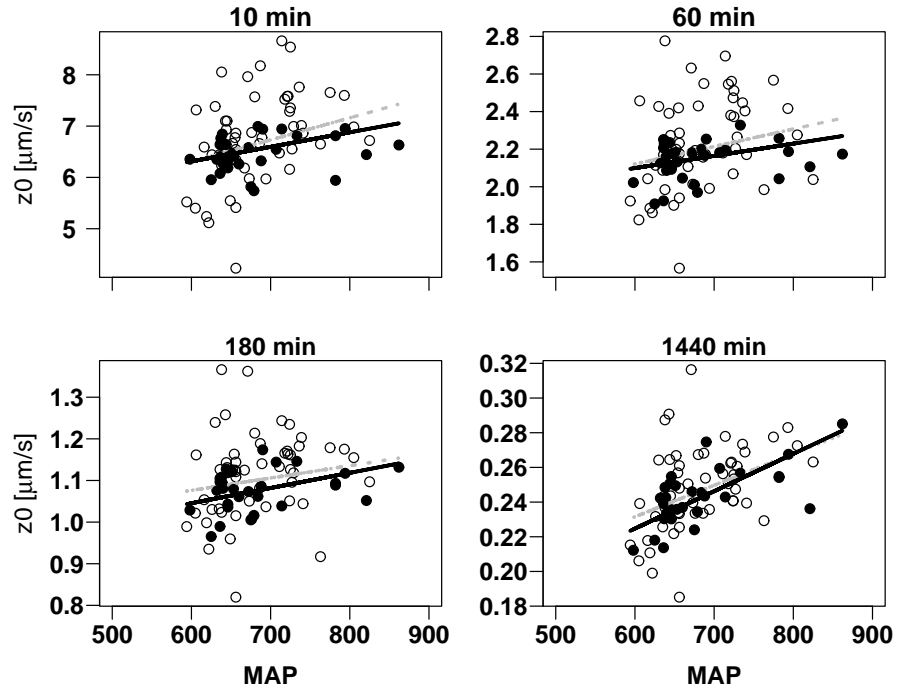


Figure 2. The OLS and GLS regression, in grey and black, respectively, showing the variation of z_0 as a function of MAP . The marked z_0 values are from stations with more than 30 years of observation and thereby have a high weight in the GLS model.

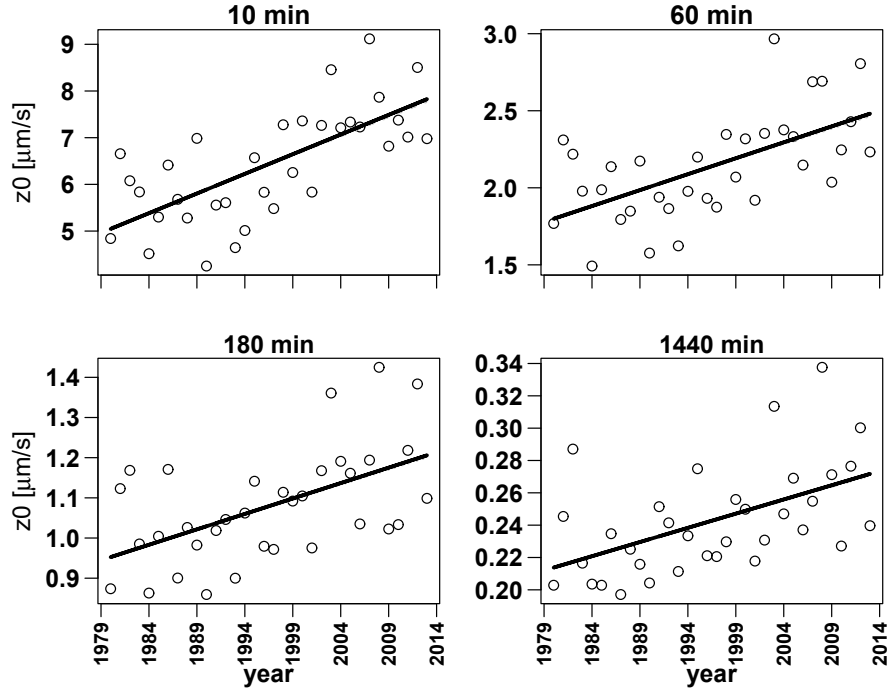


Figure 3. The OLS and GLS regression (which are identical), showing the variation of z_0 as a function of *time*.

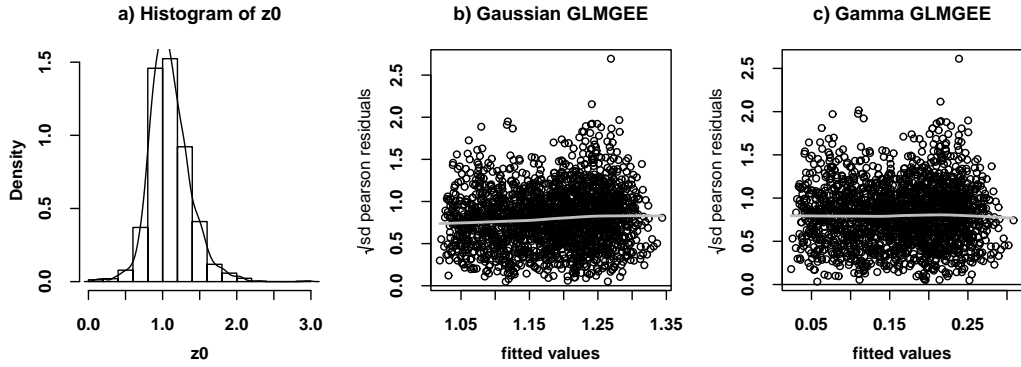


Figure 4. Evaluation of $z_{0i,s}$ to select the best underlying distribution for the GLMGEE model. b) and c) show the residuals of Eq.(18) assuming a Gaussian distribution with identity link and a Gamma distribution with log link, respectively. The graphs are for a duration of 180 minutes but similar results are obtained for the other three durations.

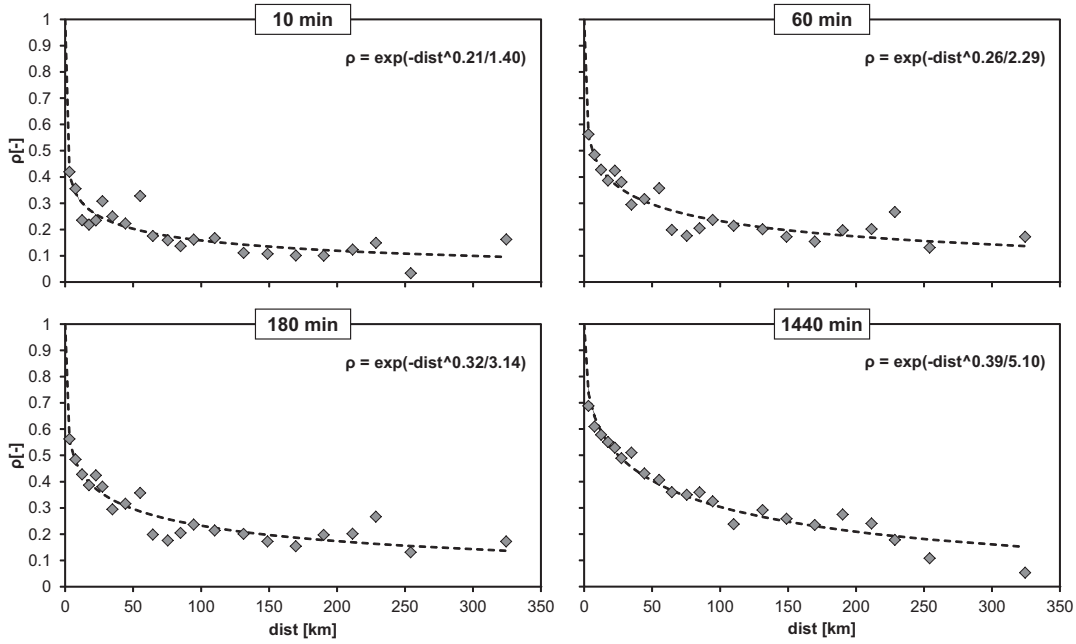


Figure 5. The correlogram for the spatial variation of λ . Blue dots represents the observed correlation averaged in selected bins, the black line represents the fitted exponential model given in the top left of each graphs.

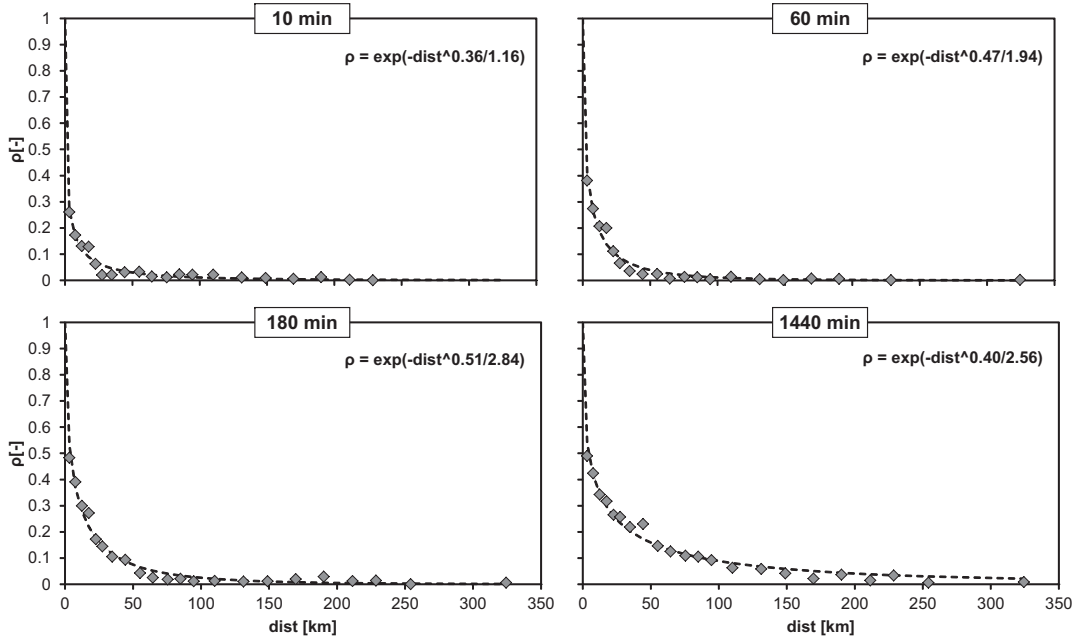


Figure 6. The correlogram for the spatial variation of μ . Blue dots represents the observed correlation averaged in selected bins, the black line represents the fitted exponential model given in the top left of each graphs.

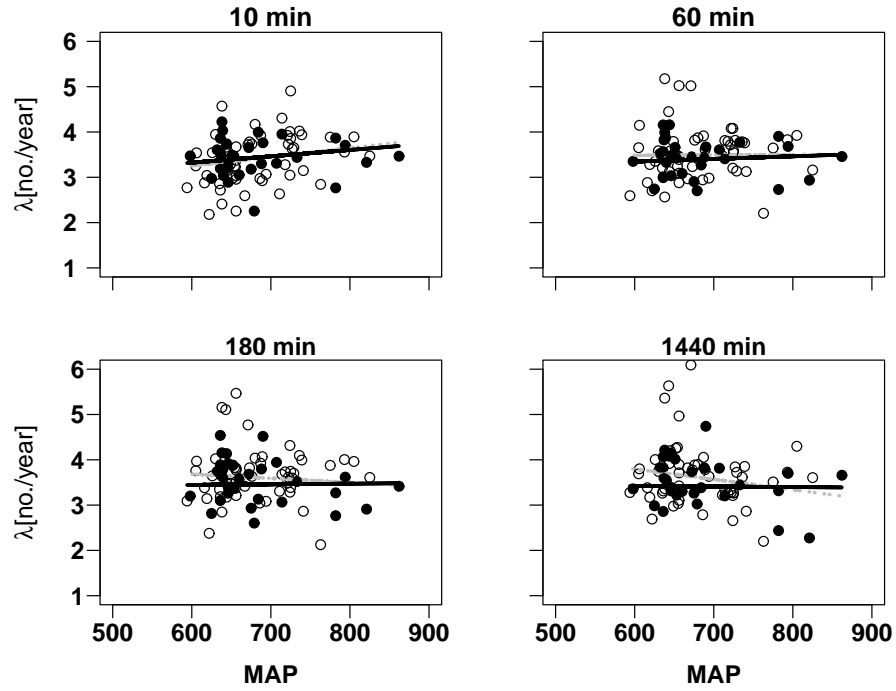


Figure 7. Marginal regional variation of λ from a PDS sampling with a spatio-temporal z_θ . OLS and GLS regression are shown in grey and black, respectively. The marked λ values are from stations with more than 30 years of observation.

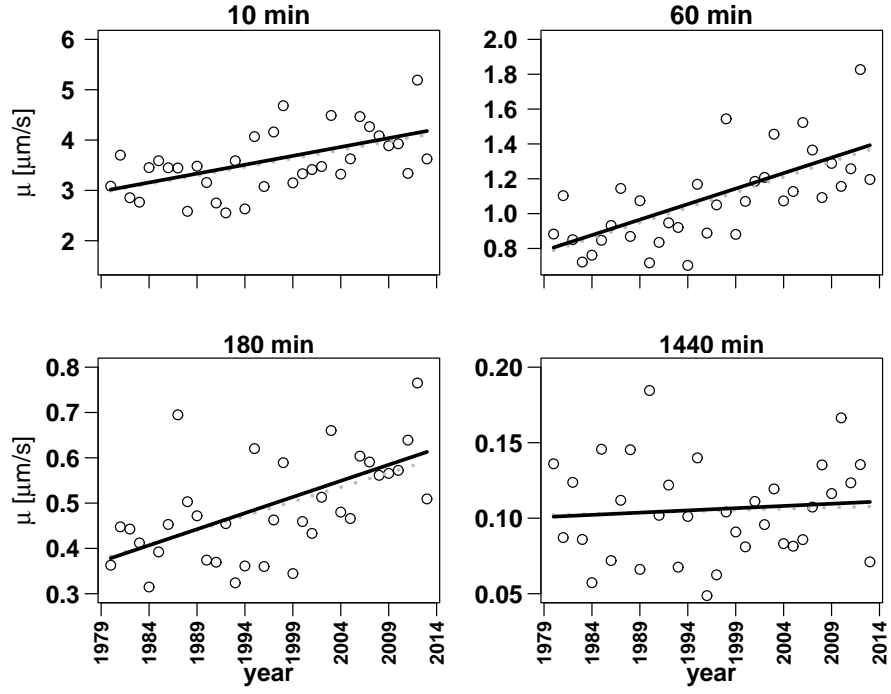


Figure 8. Marginal temporal variation of μ from a PDS sampling with a spatio-temporal z_θ . OLS (grey line) and GLS (black line) regression estimates are almost identical.

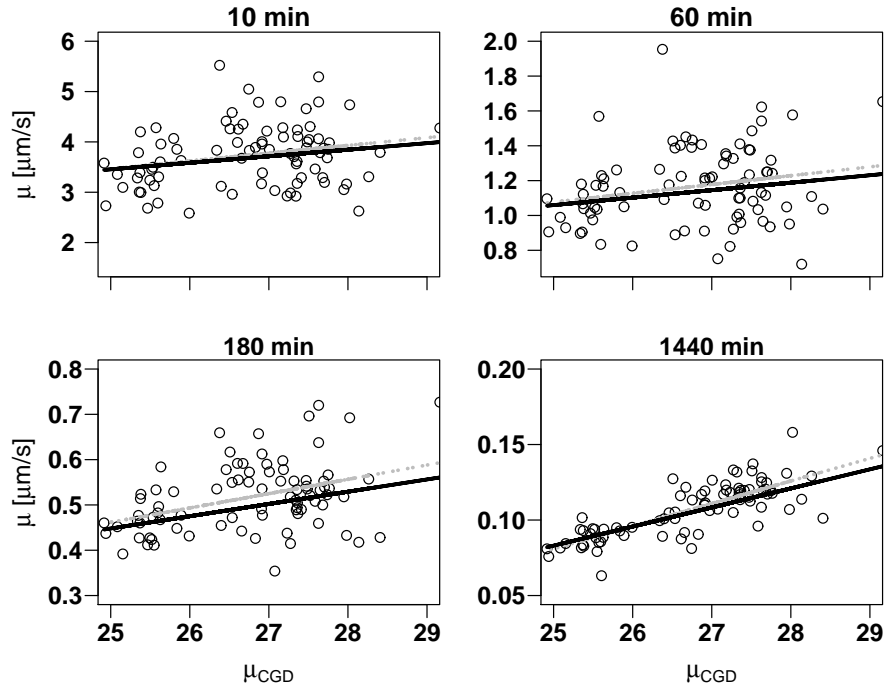


Figure 9. Marginal regional variation of μ from a PDS sampling with a spatio-temporal z_θ . OLS and GLS regression are shown in grey and black, respectively.

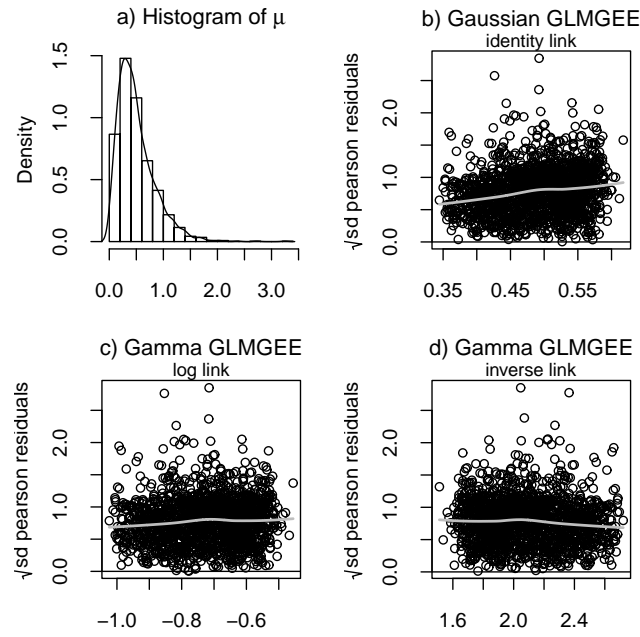


Figure 10. Evaluation of $\mu_{i,s}$ to select the best underlying distribution for the GLMGEE model. b), c) and d) show the residuals of Eq.(24) assuming a Gaussian distribution with identity link, a Gamma distribution with log link and a Gamma distribution with inverse link, respectively. The graphs are for a duration of 180 minutes.

Table 1. The estimated GLS parameters in the marginal regional regression models for z_{0s} ^a

Duration [min]	a [$\mu\text{m/s}$]	b [$\mu\text{m/s/mm}$]	p_b	$\hat{\sigma}_\delta^2$	MSE_e	MSE_e reduction
10	4.61 (1.102)	0.0028 (0.0015)	0.07	0.248	0.492 (80)	9.77%
60	1.71 (0.346)	$6.5 \cdot 10^{-4}$ ($4.8 \cdot 10^{-4}$)	0.18	0.022	0.041 (80)	5.91%
180	0.83 (0.158)	$3.6 \cdot 10^{-4}$ ($2.2 \cdot 10^{-4}$)	0.10	0.0050	0.0073 (80)	3.57%
1440	0.10 (0.037)	$2.2 \cdot 10^{-4}$ ($5.2 \cdot 10^{-5}$)	<0.01	$2.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-4}$ (80)	21.82%

^a p_b measures the significance of b , which represents the regression to MAP [mm], see Eq.(16). MSE_e denotes the Mean Squared Error of the residuals with degrees of freedom given in parenthesis. The MSE_e reduction is estimated by comparison with an intercept model. For the regression parameters standard deviation is given in parenthesis.

Table 2. The estimated GLS parameters in the marginal temporal regression models for z_{0i} ^a

Duration [min]	a [$\mu\text{m/s}$]	c [$\mu\text{m/s/year}$]	p_c	$\hat{\sigma}_\delta^2$	MSE_e	MSE_e reduction
10	4.95 (0.40)	0.085 (0.018)	<0.01	$5.4 \cdot 10^{-4}$	0.726 (31)	47.43%
60	1.78 (0.13)	0.021 (0.0059)	<0.01	$4.2 \cdot 10^{-4}$	0.073 (31)	35.07%
180	0.94 (0.060)	0.0078 (0.0027)	-	0	0.014 (31)	28.74%
1440	0.21 (0.017)	0.0018 ($7.5 \cdot 10^{-4}$)	-	0	$7.9 \cdot 10^{-4}$ (31)	26.56%

^a p_c measures the significance of c , which represents the regression to *time* [year], see Eq.(17). MSE_e denotes the Mean Squared Error of the residuals with degrees of freedom given in parenthesis. The MSE_e reduction is estimated by comparison with an intercept model. For the regression parameters standard deviation is given in parenthesis.

Table 3. The estimated parameters in the Gaussian GLMGEE models for $z_{0i,s}$, compared to the two marginal GLS models from Table 1 and Table 2^a

Duration [min]		a [$\mu\text{m/s}$]	b [$\mu\text{m/s/mm}$]	p_b	c [$\mu\text{m/s/year}$]	p_c	MSE _{ϵ} reduction
10	GLMGEE	3.79 (0.67)	0.0021 (0.0009)	0.02	0.080 (0.014)	<0.01	14.3%
	marginal GLS		0.0028 (0.0015)	0.07	0.085 (0.018)	<0.01	
60	GLMGEE	1.50 (0.23)	$5.3 \cdot 10^{-4}$ ($3.1 \cdot 10^{-4}$)	0.08	0.018 (0.005)	<0.01	10.0%
	marginal GLS		$6.5 \cdot 10^{-4}$ ($4.8 \cdot 10^{-4}$)	0.18	0.021 (0.0059)	0.01	
180	GLMGEE	0.72 (0.10)	$3.8 \cdot 10^{-4}$ ($1.5 \cdot 10^{-4}$)	<0.01	0.0060 (0.0018)	<0.01	7.10%
	marginal GLS		$3.6 \cdot 10^{-4}$ ($2.2 \cdot 10^{-4}$)	0.10	0.0078 (0.0027)	-	
1440	GLMGEE	0.044 (0.024)	$2.6 \cdot 10^{-4}$ ($3.0 \cdot 10^{-5}$)	<0.01	0.0013 ($4.5 \cdot 10^{-4}$)	<0.01	9.7%
	marginal GLS		$2.2 \cdot 10^{-4}$ ($5.2 \cdot 10^{-5}$)	<0.01	0.0018 ($7.5 \cdot 10^{-3}$)	-	

^a p_b measures the significance of b , which represents the regression to MAP [mm], see Eq.(18). p_c measures the significance of c , which represents the regression to $time$ [year], see Eq.(18). The MSE _{ϵ} reduction is estimated by comparison with an intercept model. For the regression parameters standard deviation is given in parenthesis.

Table 4. The estimated parameters in the Poisson GLMGEE models for $N_{i,s}$ with a log-link, see Eq. (21)^a

	10 min	60 min	180 min	1440 min
a	-4.65 (0.048)	-4.63 (0.054)	-4.60 (0.052)	-4.62 (0.055)
scale of Poisson	1.22	1.26	1.23	1.15
λ [no./year]	3.48 (0.17)	3.56 (0.19)	3.67 (0.19)	3.58 (0.20)

^a a represents the model intercept, see Eq.(21). The scale measures the dispersion of the Poisson distribution, see *Gregersen et al.* [2013a]. λ is predicted from the model using an observation period of one year ($l_{i,s} = 365$ days). Uncertainty on the estimates is given in parenthesis.

Table 5. The estimated parameters in the Gamma GLMGEE models for $\mu_{0i,s}$, compared to the two marginal GLS models^a

Duration [min]		a	b	p_b	c	p_c	Gamma scale	MSE _{ϵ} reduction
10	GLMGEE	0.51 (0.19)	-7.0·10 ⁻³ (7.1·10 ⁻³)	0.32	-2.4·10 ⁻³ (5.8·10 ⁻⁴)	<0.01	0.49	
	marginal GLS		0.13 (0.079)	0.10	0.035 (9.8·10 ⁻³)	<0.01		
	reduced GLMGEE	0.32 (0.016)			-2.4·10 ⁻³ (5.8·10 ⁻⁴)	<0.01	0.49	3.07%
60	GLMGEE	1.24 (0.64)	-0.020 (0.024)	0.93	-0.014 (2.6·10 ⁻³)	<0.01	0.53	
	marginal GLS		0.043 (0.031)	0.17	0.018 (3.7·10 ⁻³)	<0.01		
	reduced GLMGEE	1.19 (0.064)			-0.014 (2.6·10 ⁻³)	<0.01	0.53	9.64%
180	GLMGEE	4.83 (1.32)	-0.084 (0.051)	0.10	-0.026 (6.0·10 ⁻³)	<0.01	0.49	7.99%
	marginal GLS		0.027 (0.012)	0.02	7.1·10 ⁻³ (1.7·10 ⁻³)	<0.01		
	reduced GLMGEE	2.61 (0.15)			-0.026 (5.9·10 ⁻³)	<0.01	0.50	5.64%
1440	GLMGEE	47.77 (7.76)	-1.39 (0.29)	<0.01	-0.019 (0.052)	0.71	0.53	
	marginal GLS		0.013 (2.6·10 ⁻³)	<0.01	2.9·10 ⁻⁴ (5.6·10 ⁻⁴)	0.60		
	reduced GLMGEE	47.65 (7.79)	-1.40 (0.30)	<0.01			0.53	17.8%

^aDue to the inverse link function of the GLMGEE model, parameters cannot be directly compared but the significance can. In the reduced GLMGEE non-significant terms have been excluded. Uncertainty on the parameter estimates is given in parenthesis. MSE _{ϵ} denotes the Mean Squared Error of the residuals. The MSE _{ϵ} reduction is estimated by comparison with an intercept model.

Table 6. Estimations of L_{cv} and κ^a

	10 min	60 min	180 min	1440 min
L_{cv}	0.53 ($7.1 \cdot 10^{-5}$)	0.55 ($2.7 \cdot 10^{-4}$)	0.54 ($4.0 \cdot 10^{-4}$)	0.54 ($1.5 \cdot 10^{-4}$)
κ	-0.12 ($8.8 \cdot 10^{-4}$)	-0.19 ($2.9 \cdot 10^{-3}$)	-0.16 ($4.6 \cdot 10^{-3}$)	-0.15 ($1.8 \cdot 10^{-3}$)

^aUncertainty on the parameter estimates is given in parenthesis.

Table 7. Estimations of design intensities with a return period of 10 years, uncertainty is given in parenthesis^a

	MAP [mm]	μ_{CGD} [mm]	10 min [$\mu\text{m/s}$]	60 min [$\mu\text{m/s}$]	180 min [$\mu\text{m/s}$]	1440 min [$\mu\text{m/s}$]
Madsen <i>et al.</i> (in prep)	707	25.5	21.30 (1.83)	6.91 (0.94)	2.99 (0.35)	0.61 (0.052)
Eq.(25) with i=1996	707	25.5	20.00 (0.90)	6.59 (0.50)	2.93 (0.26)	0.59 (0.032)
Eq.(25) with i=2005	707	25.5	21.90 (0.97)	7.42 (0.58)	3.20 (0.29)	0.60 (0.032)
Madsen <i>et al.</i> (in prep)	660	27.4	20.99 (1.81)	6.84 (0.93)	3.21 (0.38)	0.69 (0.062)
Eq.(25) with i=1996	660	27.4	19.93 (0.90)	6.57 (0.50)	3.05 (0.28)	0.68 (0.045)
Eq.(25) with i=2005	660	27.4	21.84 (0.97)	7.39 (0.58)	3.36 (0.31)	0.69 (0.045)

^aThe selected locations correspond to the two largest cities in Denmark (Aarhus and Copenhagen).